6. (a) This is the final tableau since there are no negative entries in the objective row. The maximum value is $P = 140$. It occurs when $x_1 = 20$ and $x_2 = 20$.

8. (b) This tableau requires further pivoting since there are still negative entries in the objective row. We find the pivot element by selecting the first column containing a negative entry in the objective row. Here it is column x_1. We select the pivot row by computing the quotients formed by dividing the entry in the right hand side by the corresponding positive entry of the pivot column.
The pivot row has the smallest nonnegative quotient.

$$10 \div \frac{1}{4} = 40 \text{ and } 8 \div \frac{7}{4} = \frac{32}{7}$$

The new pivot element is $\frac{1}{4}$ in row s_1, column x_1.

9. (c) There is no solution to this problem. Although there is a negative entry in the objective row, all entries in the pivot column are negative, so the problem is unbounded and has no solution.

14. To solve the problem using the simplex method, we first must introduce slack variables and construct the initial tableau.

Maximize

$$P - x_1 - 5x_2 = 0$$

subject to the constraints

$$\begin{aligned} 2x_1 + x_2 + s_1 \quad &= 10 \\ x_1 + 2x_2 \quad + s_2 &= 10 \\ x_1 \ge 0 \quad x_2 \ge 0 \quad s_1 \ge 0 \quad s_2 &\ge 0 \end{aligned}$$

The initial Simplex tableau with the pivot element marked is below. The first negative entry in the objective row identifies the pivot column. The smallest nonnegative quotient formed by the RHS and positive entries in the pivot column identifies the pivot row. Here the pivot element is in row s_1, column x_1.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & \boxed{2} & 1 & 1 & 0 & 10 \\ 0 & 1 & 2 & 0 & 1 & 10 \\ \hline 1 & -1 & -5 & 0 & 0 & 0 \end{array}\right] \end{array}$$

We pivot, first by dividing to make the pivot element 1, and then by using row operations to make the other entries in the pivot column zero.

$$\xrightarrow{R_1=\frac{1}{2}r_1}\begin{array}{c} \text{BV} \\ x_1 \\ s_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & & 5 \\ 0 & 1 & 2 & 0 & 1 & & 10 \\ \hline 1 & -1 & -5 & 0 & 0 & & 0 \end{array}\xrightarrow[R_3=R_1+r_3]{R_2=-R_1+r_2}\begin{array}{c} \text{BV} \\ x_1 \\ s_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & & 5 \\ 0 & 0 & \boxed{\frac{3}{2}} & -\frac{1}{2} & 1 & & 5 \\ \hline 1 & 0 & -\frac{9}{2} & \frac{1}{2} & 0 & & 5 \end{array}$$

Since there is still a negative entry in the objective row, we need to pivot again. We choose the pivot entry as before. The pivot column will be x_2, the pivot row will be s_2.

$$\frac{5}{\frac{3}{2}}=\frac{10}{3}<\frac{5}{\frac{1}{2}}=10$$

$$\xrightarrow{R_2=\frac{2}{3}r_2}\begin{array}{c} \text{BV} \\ x_1 \\ x_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & 1 & \frac{1}{2} & \boxed{\frac{1}{2}} & 0 & & 5 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & & \frac{10}{3} \\ \hline 1 & 0 & -\frac{9}{2} & \frac{1}{2} & 0 & & 5 \end{array}\xrightarrow[R_3=\frac{9}{2}R_2+r_3]{R_1=-\frac{1}{2}R_2+r_1}\begin{array}{c} \text{BV} \\ x_1 \\ x_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & 1 & 0 & \boxed{\frac{2}{3}} & -\frac{1}{3} & & \frac{10}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & & \frac{10}{3} \\ \hline 1 & 0 & 0 & -1 & 3 & & 20 \end{array}$$

Since there is still a negative entry in the objective row, we pivot another time. The pivot column is s_1, and the pivot row is x_1 (the only nonnegative entry in the pivot column).

$$\xrightarrow{R_1=\frac{3}{2}r_1}\begin{array}{c} \text{BV} \\ s_1 \\ x_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & \frac{3}{2} & 0 & 1 & -\frac{1}{2} & & 5 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & & \frac{10}{3} \\ \hline 1 & 0 & 0 & -1 & 3 & & 20 \end{array}\xrightarrow[R_3=R_1+r_3]{R_2=\frac{1}{3}R_1+r_2}\begin{array}{c} \text{BV} \\ s_1 \\ x_2 \\ P \end{array}\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & & \text{RHS} \\ \hline 0 & \frac{3}{2} & 0 & 1 & -\frac{1}{2} & & 5 \\ 0 & \frac{1}{2} & 1 & 0 & \frac{1}{2} & & 5 \\ \hline 1 & \frac{3}{2} & 0 & 0 & \frac{5}{2} & & 25 \end{array}$$

This is the final tableau since all entries in the objective row are nonnegative. The solution is $P = 25$, obtained when $x_1 = 0$ and $x_2 = 5$.

15. To solve the problem using the simplex method, we first must introduce slack variables and construct the initial tableau.

Maximize

$$P-5x_1-7x_2=0$$

subject to the constraints

$$\begin{aligned} x_1+2x_2+s_1 \quad &=2 \\ 2x_1+\ x_2 \quad +s_2 &=2 \\ x_1\ge 0\ x_2\ge 0\ s_1\ge 0\ s_2 &\ge 0 \end{aligned}$$

The initial Simplex tableau with the pivot element marked is below. The first negative entry in the objective row identifies the pivot column. The smallest nonnegative quotient formed by the RHS and positive entries in the pivot column identifies the pivot row. Here the pivot element is in row s_2, column x_1. We then use row operations to make the pivot element 1 and all the other entries in the pivot column 0.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 2 & 1 & 0 & 2 \\ 0 & \boxed{2} & 1 & 0 & 1 & 2 \\ \hline 1 & -5 & -7 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow[\substack{R_1=-R_2+r_1 \\ R_3=5R_2+r_3}]{R_2=\frac{1}{2}r_2} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 0 & \boxed{\frac{3}{2}} & 1 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \hline 0 & 0 & -\frac{9}{2} & 0 & \frac{5}{2} & 5 \end{array}\right] \end{array}$$

Since there is still a negative entry in the objective row, we need to pivot again. We choose the pivot entry as before. The pivot column will be x_2, the pivot row will be s_1 because

$$\frac{1}{\frac{3}{2}} = \frac{2}{3} < \frac{1}{\frac{1}{2}} = 2$$

$$\xrightarrow[\substack{R_2=-\frac{1}{2}R_1+r_2 \\ R_3=\frac{9}{2}R_1+r_3}]{R_1=\frac{2}{3}r_1} \begin{array}{c} \text{BV} \\ x_2 \\ x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \hline 0 & 0 & 0 & 3 & 1 & 8 \end{array}\right] \end{array}$$

This is the final tableau since all entries in the objective row are nonnegative. The solution is $P = 8$, obtained when $x_1 = \frac{2}{3}$ and $x_2 = \frac{2}{3}$.

19. To solve the problem using the simplex method, we first must introduce slack variables and construct the initial tableau.

Maximize

$$P - 2x_1 - x_2 - x_3 = 0$$

subject to the constraints

$$\begin{aligned} -2x_1 + x_2 - 2x_3 + s_1 \quad &= 4 \\ x_1 - 2x_2 + x_3 \quad + s_2 &= 2 \\ x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0 \quad s_1 \ge 0 \quad s_2 &\ge 0 \end{aligned}$$

The initial Simplex tableau with the pivot element marked is below. The first negative entry in the objective row identifies the pivot column. The smallest nonnegative quotient formed by the RHS and positive entries in the pivot column identifies the pivot row. Here the pivot element is in row s_2, column x_1. We then use row operations to make all entries in the pivot column other than the pivot element 0.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} P & x_1 & x_2 & x_3 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & -2 & 1 & -2 & 1 & 0 & 4 \\ 0 & \boxed{1} & -2 & 1 & 0 & 1 & 2 \\ \hline 1 & -2 & -1 & -1 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow[R_3=2R_2+r_3]{R_1=2R_2+r_1} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} P & x_1 & x_2 & x_3 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 0 & -3 & 0 & 1 & 2 & 8 \\ 0 & 1 & -2 & 1 & 0 & 1 & 2 \\ \hline 1 & 0 & -5 & 1 & 0 & 2 & 4 \end{array}\right] \end{array}$$

The new pivot column is x_2, since –5 is the only negative entry in the objective row. But all the entries in the pivot column are negative. This problem is unbounded and has no solution.