7. Evaluate the objective function at each corner point; then choose the maximum and minimum values.

$z = x + y$

Point: (2, 2)	Point: (8, 1)	Point: (2, 7)	Point: (7, 8)
$z = 2 + 2$	$z = 8 + 1$	$z = 2 + 7$	$z = 7 + 8$
$z = 4$	$z = 9$	$z = 9$	$z = 15$

Maximum: $z = 15$; Minimum: $z = 4$

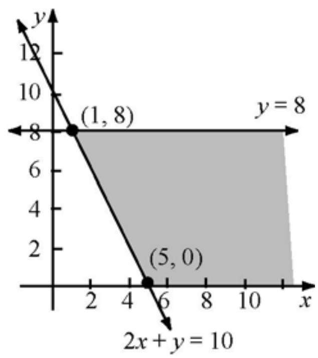
12. Evaluate the objective function at each corner point; then choose the maximum and minimum values.

$z = 4x + 3y$

Point: (2, 2)	Point: (8, 1)	Point: (2, 7)	Point: (7, 8)
$z = 4(2) + 3(2)$	$z = 4(8) + 3(1)$	$z = 4(2) + 3(7)$	$z = 4(7) + 3(8)$
$z = 14$	$z = 35$	$z = 29$	$z = 52$

Maximum: $z = 52$; Minimum: $z = 14$

18. The constraints form the unbounded region that is shaded in the graph.



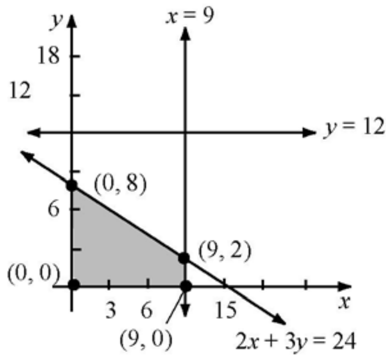
One corner point is (5, 0).
The second corner, (1, 8) was found by solving

$$\begin{cases} y = 8 \\ 2x + y = 10 \end{cases}$$

Substituting $y = 8$ into the second equation, we find:

$$\begin{aligned} 2x + 8 &= 10 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

20. The constraints form the region that is shaded in the graph.



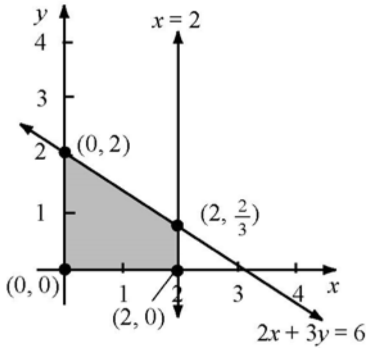
Three of the corner points (0, 0), (9, 0), and (0, 8) are easy to identify. The fifth corner, (9, 2) was found by solving

$$\begin{cases} x = 9 \\ 2x + 3y = 24 \end{cases}$$

Substituting $x = 9$ into the second equation, we find

$$\begin{aligned} 2 \cdot 9 + 3y &= 24 \\ 18 + 3y &= 24 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

22. To maximize $z = 5x + 7y$, graph the system of linear inequalities, shade the set of feasible points, and locate the corner points. Then evaluate the objective function at each corner point.



The corner points are (0, 0), (2, 0), (0, 2) and $\left(2, \frac{2}{3}\right)$.

The third point was obtained by solving

$$\begin{cases} 2x + 3y = 6 \\ x = 2 \end{cases}$$

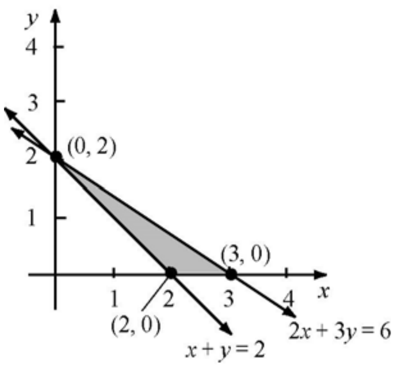
Substituting $x = 2$ into the first equation we find

$$2(2) + 3y = 6 \text{ or } y = \frac{2}{3}.$$

Corner Point (x, y)	Value of the Objective Function $z = 5x + 7y$
(0, 0)	$z = 5(0) + 7(0) = 0$
(2, 0)	$z = 5(2) + 7(0) = 10$
(0, 2)	$z = 5(0) + 7(2) = 14$
$\left(2, \frac{2}{3}\right)$	$z = 5(2) + 7\left(\frac{2}{3}\right) = \frac{44}{3}$

The maximum value of z is $\frac{44}{3}$, and it occurs at the point $\left(2, \frac{2}{3}\right)$.

23. To maximize $z = 5x + 7y$, graph the system of linear inequalities, shade the set of feasible points, and locate the corner points. Then evaluate the objective function at each corner point.

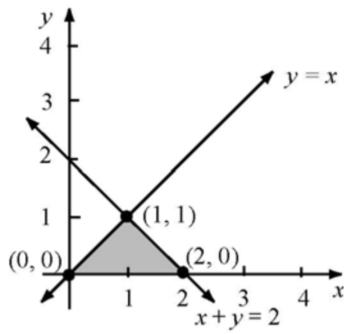


The corner points are (2, 0), (3, 0), and (0, 2).

Corner Point (x, y)	Value of the Objective Function $z = 5x + 7y$
(2, 0)	$z = 5(2) + 7(0) = 10$
(3, 0)	$z = 5(3) + 7(0) = 15$
(0, 2)	$z = 5(0) + 7(2) = 14$

The maximum value of z is 15, and it occurs at the point (3, 0).

29. To minimize $z = 2x + 3y$, graph the system of linear inequalities, shade the set of feasible points, and locate the corner points. Then evaluate the objective function at each corner point.



The corner points are (0, 0), (2, 0), and (1, 1). The third point was obtained by solving

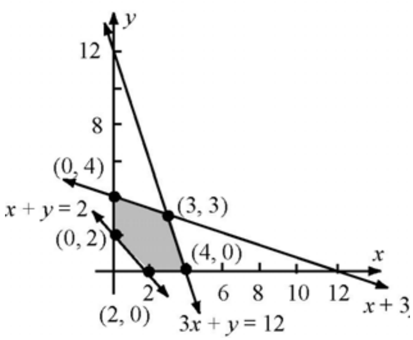
$$\begin{cases} x + y = 2 \\ \quad y = x \end{cases}$$

Substituting x for y in equation 1, we find $2x = 2$, or $x = 1$. Back-substituting in equation 2, gives $y = 1$.

Corner Point (x, y)	Value of the Objective Function $z = 2x + 3y$
(0, 0)	$z = 2(0) + 3(0) = 0$
(2, 0)	$z = 2(2) + 3(0) = 4$
(1, 1)	$z = 2(1) + 3(1) = 5$

The minimum value of z is 0, and it occurs at the point (0, 0).

31. To minimize $z = 2x + 3y$, graph the system of linear inequalities, shade the set of feasible points, and locate the corner points. Then evaluate the objective function at each corner point.



The corner points are (2, 0), (4, 0), (0, 2), (0, 4), and (3, 3). The fifth point was obtained by solving

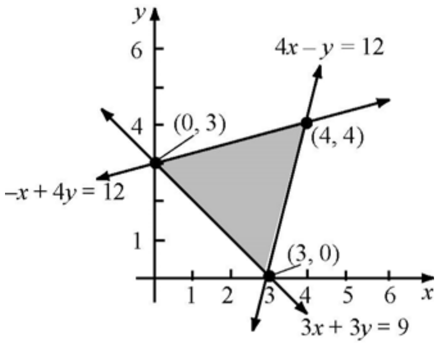
$$\begin{cases} 3x + y = 12 \\ x + 3y = 12 \end{cases}$$

Subtracting the first equation from three times the second equation gives, $8y = 24$ or $y = 3$. Back-substituting 3 for y in the first equation gives $3x + 3 = 12$ or $3x = 9$ or $x = 3$.

Corner Point (x, y)	Value of the Objective Function $z = 2x + 3y$
(2, 0)	$z = 2(2) + 3(0) = 4$
(4, 0)	$z = 2(4) + 3(0) = 8$
(0, 2)	$z = 2(0) + 3(2) = 6$
(0, 4)	$z = 2(0) + 3(4) = 12$
(3, 3)	$z = 2(3) + 3(3) = 15$

The minimum value of z is 4, and it occurs at the point (2, 0).

45. To find the maximum and minimum values of z, we graph the constraints, shade the set of feasible points, find the corner points, and evaluate the objective function, z, at each corner point.



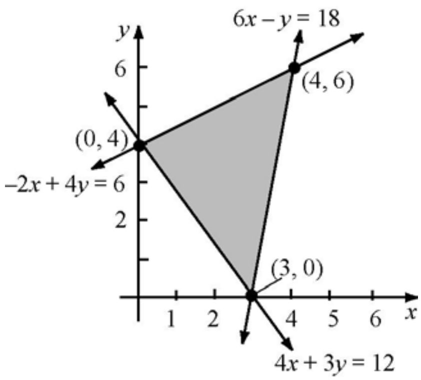
The corner points are (3, 0), (0, 3), and (4, 4). We find (4, 4) by solving $\begin{cases} -x+4y=12 & (1) \\ 4x-\ \ y=12 & (2) \end{cases}$.

Multiplying (1) by 4 and adding it to (2) gives $15y = 60$ or $y = 4$. Back-substituting into (2) gives $4x = 16$ or $x = 4$.

Corner Point	$z = 18x + 30y$
(3, 0)	$z = 18(3) + 30(0) = 54$
(0, 3)	$z = 18(0) + 30(3) = 90$
(4, 4)	$z = 18(4) + 30(4) = 192$

The maximum value of $z = 192$ at the point (4, 4).
The minimum value of $z = 54$ at the point (3, 0).

46. To find the maximum and minimum values of z, we will graph the constraints, shade the set of feasible points, find the corner points, and evaluate the objective function, z, at each corner point.



The corner points are (3, 0), (0, 4), and (4, 6).
We find (4, 6) by solving

$$\begin{cases} -2x+4y=16 & (2) \\ 6x-\ \ y=18 & (3) \end{cases}$$

Multiplying (2) by 3 and adding it to (3) gives $11y = 66$ or $y = 6$. Back-substituting into (3) gives $6x = 24$ or $x = 4$.

Corner Point	$z = 20x + 16y$
(3, 0)	$z = 20(3) + 16(0) = 60$
(0, 4)	$z = 20(0) + 16(4) = 64$
(4, 6)	$z = 20(4) + 16(6) = 176$

The maximum value of z = 176 at the point (4, 6).
The minimum value of $z = 60$ at the point (3, 0).